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Performance analysis of an elevator system during up-peak

Yutae Lee[a,*], Tai Suk Kim[b], Ho-Shin Cho[c], Dan Keun Sung[d], Bong Dae Choi[e]

[a] *Department of Information and Communication Engineering, Dongeui University, Busan 614-714, Republic of Korea*
[b] *Samsung Electronics Co., LTD., Suwon, Gyeonggi 442-742, Republic of Korea*
[c] *School of Electrical Engineering and Computer Science, Kyungpook National University, Daegu 702-701, Republic of Korea*
[d] *Department of Electrical Engineering, KAIST, Taejon 305-701, Republic of Korea*
[e] *Department of Mathematics, Korea University, Seoul 136-701, Republic of Korea*

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A B S T R A C T

We model and analyze an elevator system during up-peak. We study the round-trip time, whose distribution depends strongly on the number of passengers waiting at the lobby, as well as the number of stops and the highest reversal floor. The distribution functions of the passenger queue length in the lobby, the round-trip time, the waiting time, the ride time and the journey time are derived.

1. Introduction

Nowadays, high-rise buildings are normally serviced by elevators, as the vertical traffic transport equipment. The parameters such as the elevator capacity and running speed must be confirmed during the building design. This requires that we analyze the elevator traffic reasonably [1,2]. Studies on elevator system in a high-rise building have been performed by [1–6]. The analysis of elevator traffic has mostly concentrated on up-peak, since it is the worst traffic situation from the elevator capacity point of view [7]. During up-peak, passengers arrive randomly at an entrance floor (lobby), and randomly choose one of the higher floors as their destination. The elevator moves from the lobby, up to all the destination floors of the passengers inside the elevator, stops when it arrives at the highest destination floor, and goes back to the lobby only when new passengers are present there.

In designing a building, the usual criterion for deciding the capacity of elevators, is that passenger's waiting time should not exceed some specified value during up-peak. Another criterion used in elevator design is that one should minimize some weighted sum of waiting time and riding time, since time spent riding a crowded elevator is at least as onerous as time spent waiting in a lobby [8]. If a passenger's objective is simply to reach his destination as soon as possible, he would give equal weight to waiting or riding time.

Until now, there are few papers to analyze, mathematically, the performance of an elevator during up-peak. Siikoken [7] discussed passenger service in an elevator system during up-peak. She derived the mean values of the round-trip time and the passenger ride time. Assuming that the round-trip times follow i.i.d. Erlangian distributions, and the elevator is always busy, she derived the mean values of the passenger waiting time and the passenger journey time. Newell [8] derived the mean value of the journey time for various strategies in order to compare their performance. While most of the analyses in

* Corresponding author.
E-mail address: ylee@deu.ac.kr (Y. Lee).

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the literature [7–11] have derived only the mean values of the performance measures during up-peak, this paper obtains their distributions.

In this paper, we model and analyze an elevator system during up-peak, where passengers arrive randomly at a lobby according to a Poisson process. According to [12], the Poisson arrival assumption at least cannot be rejected [7]. Inter-floor travel times and stop times at a floor (including accelerating, decelerating, opening and closing of the door) are taken as a constant. Only up-peak traffic is considered. The elevator has finite capacity, and passengers arriving in the lobby are served in the order of arrival. To analyze several performance measures of the elevator model, a discrete-time Markov chain (DTMC) is embedded at departure epochs from the lobby, arrival epochs at a highest destination floor, and departure epochs from such a floor back to the lobby. By including the number of stops between two consecutive Markov points, a discrete-time semi-Markov sequence is introduced for which the semi-Markov kernel can be calculated. Knowing this kernel, the one-step transition probabilities of the DTMC are calculated and the steady-state distribution of the DTMC is found using well-known methods for M/G/1 type queues. Having available the steady-state distribution, we continue to study the round-trip time of the elevator, the passenger waiting time at the lobby, the passenger ride time, and the passenger journey time. For the round-trip time we derive the distribution function, using the semi-Markov kernel and the steady-state distribution of the DTMC. For the passenger waiting time, we derive the Laplace–Stieltjes Transform (LST), considering a virtual passenger and using a series of conditioning and unconditioning arguments. To find the distribution of the passenger ride time, we use the distribution of the number of passengers in the elevator during the ride of the virtual passenger. Finally, the LST of the passenger journey time is found by a series of conditioning/unconditioning arguments. Numerical examples are presented for an elevator system. We present figures of the probability mass function of the number of passengers in the elevator system (for different values of the passenger arrival rate) at departure epoch from the lobby, the cumulative distribution functions of the round-trip time, and (in one figure) the mean round-trip time, ride time and journey time as a function of the arrival rate.

2. Analysis of elevator system

2.1. Description of model

Let N be the number of floors above the lobby, t_r the run time of an elevator for one floor at the nominal speed, t_l the stop time of the elevator at the lobby and t_s the stop time at the other floors. Each stop time includes accelerating, decelerating, transferring of passengers, and the opening and closing of the door. It is assumed that t_r, t_l and t_s are all constants.

During up-peak, passengers are assumed to arrive at the lobby according to a Poisson process with arrival rate λ, and no passengers arrive at the other floors. Hence, there are no stops from the highest reversal floor to the lobby. A single elevator serves the waiting passengers at the lobby by FCFS (First-Come First-Served). The elevator can serve up to c passengers at the same time. The destination floor of each passenger is uniformly distributed among all the floors. If there are no passengers waiting at the lobby when the elevator arrives at the highest reversal floor, then the elevator stays idle at the floor and is activated by the first passenger arrival at the lobby; if there are passengers waiting at the lobby, the elevator is returned to the lobby.

2.2. Number of passengers in the elevator system

The elevator system is embedded at departure epochs from the lobby, arrival epochs at a highest destination floor, and departure epochs from such a floor back to the lobby. We define by L_n the total number of passengers in the elevator system at the nth embedded point, and by F_n the location of the elevator at the nth embedded point. Let S_n be the number of stops between the $(n-1)$st and the nth embedded points.

It follows from the properties of the Poisson process that $\{(L_n, F_n), n = 1, 2, \ldots\}$ forms a Markov chain with state space $\{(i, j), i = 0, 1, \ldots, j = 0, 1, \ldots, N, i + j > 0\}$. Since the random variables $S_n, n = 1, 2, \ldots$, are conditionally independent under the condition that $\{(L_n, F_n), n = 1, 2, \ldots\}$ is given, the sequence of trivariate $\{(L_n, F_n, S_n), n = 1, 2, \ldots\}$ forms a discrete-time semi-Markov sequence.

The semi-Markov process is characterized by the semi-Markov kernel, defined as

$$Q_{(i,j),(k,l)}(m) \equiv P\{L_n = k, F_n = l, S_n = m | L_{n-1} = i, F_{n-1} = j\},$$
$$i, k = 0, 1, \ldots, j, l, m = 0, 1, \ldots, N, i + j > 0, \quad k + l > 0.$$

To derive the semi-Markov kernel $Q_{(i,j)(k,l)}(m)$, we define the conditional probability $G_i(l, m)$, $1 \le i \le c$, $1 \le l \le N$, $1 \le m \le (i, l)^-$, that the elevator stops m times before the up-trip is ended at the highest reversal floor l, given that the elevator leaves the lobby with i passengers. Finding an expression $G_i(l, m)$ requires the calculation of the probability that, in an experiment of throwing i numbered balls randomly into N boxes, at least one ball ends up in box l, no ball ends up in any of the boxes $l + 1, \ldots, N$, and exactly $m - 1$ boxes of the boxes $1, \ldots, l - 1$ will end up non-empty. We can obtain the conditional probability $G_i(l, m)$ as follow:

$$G_i(l, m) = \frac{1}{N^i} \binom{l-1}{m-1} \sum_{\substack{g_1+\cdots+g_m=i \\ 1\le g_1,\ldots,g_m\le i-m+1}} \prod_{e=1}^{m} \binom{i - \sum_{a=1}^{e-1} g_a}{g_e}, \tag{1}$$

where

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}.$$

The semi-Markov kernel $Q_{(i,j),(k,l)}(m)$ is given by

$$Q_{(i,j),(k,l)}(m) = \begin{cases} G_{(i,c)^-}(l,m)[\lambda(lt_r+mt_s)]^{k-(i-c)^+}\mathrm{e}^{-\lambda(lt_r+mt_s)}/(k-(i-c)^+)! \\ \quad \text{if } i \geq 1, j=0, k \geq (i-c)^+, 1 \leq l \leq N, 1 \leq m \leq (i,l,c)^-, \\ [\lambda(t_l+jt_r)]^{k-i}\mathrm{e}^{-\lambda(t_l+jt_r)}/(k-i)! \quad \text{if } i \geq 1, 1 \leq j \leq N, k \geq i, l=0, m=1, \\ 1 \quad \text{if } i=0, 1 \leq j \leq N, k=1, l=j, m=0, \\ 0 \quad \text{otherwise,} \end{cases}$$

where $(a_1, a_2, \ldots, a_n)^-$ indicates the minimum of $a_1, a_2, \ldots, a_n$ and $(a_1, a_2, \ldots, a_n)^+$ indicates the maximum of $0, a_1, a_2, \ldots, a_n$.

The Markov chain $\{(L_n, F_n), n = 1, 2, \ldots\}$ is irreducible and aperiodic with one-step transition matrix $P = \{P_{(i,j),(k,l)}\}$:

$$P_{(i,j),(k,l)} = \sum_{m=0}^{(i,l,c)^-} Q_{(i,j),(k,l)}(m). \tag{2}$$

Let $\{\pi_{i,j}\}$ be the steady-state distribution of the Markov chain $\{(L_n, F_n), n = 1, 2, \ldots\}$. The steady-state distribution $\{\pi_{i,j}\}$ is found by the similar argument in [13] where the steady-state distribution in an M/G/1 type queue is derived.

2.3. Elevator round-trip time

The time from the departure from the lobby until the next arrival at the lobby is called the round-trip time. We will now derive the distribution function of the round-trip time. Denoting the round-trip time by R, we define

$$R(x) \equiv P\{R \leq x\}$$

for all non-negative real number x. With probability $\pi_{i,0}/\sum_{j=1}^{\infty}\pi_{j,0}$, $i = 1, 2, \ldots$, the round-trip begins with i passengers at the lobby. We call the round-trip beginning with i passengers at the lobby the type-i round-trip, where $i = 1, 2, \ldots, c+1$. The distribution function of the type-i round-trip time R_i, $i = 1, 2, \ldots, c+1$, is denoted by $R_i(x)$, and its discrete and continuous parts by $R_i^d(x)$ and $R_i^c(x)$, respectively, with $R_{c+1}^c(x) = 0$, i.e., $R_{c+1}(x) = R_{c+1}^d(x)$.

Then, the probability distribution function of the type-i round-trip time is given by

$$R_i^d(x) = \sum_{k=1}^{\infty}\sum_{l=1}^{N}\sum_{m=1}^{\lceil (x-2lt_r-t_l)/t_s \rceil} Q_{(i,0),(k,l)}(m),$$

$$R_i^c(x) = \sum_{l=1}^{N}\sum_{m=1}^{\lceil (x-2lt_r-t_l)/t_s \rceil} Q_{(i,0),(0,l)}(m)\left[1-\mathrm{e}^{-\lambda(x-mt_s-2lt_r-t_l)}\right],$$

where $\lceil x \rceil$ is the largest integer less than or equal to x. Thus, we can write

$$\begin{aligned} R(x) &= \left[\sum_{i=1}^{\infty}\pi_{i,0}R_{(i,c+1)^-}(x)\right] \Big/ \sum_{j=1}^{\infty}\pi_{j,0} \\ &= \left[R_{c+1}(x) + \sum_{i=1}^{c}\pi_{i,0}\{R_i(x) - R_{c+1}(x)\}\right] \Big/ \sum_{j=1}^{\infty}\pi_{j,0} \\ &= \left[R_{c+1}^d(x) + \sum_{i=1}^{c}\pi_{i,0}\left\{R_i^d(x) + R_i^c(x) - R_{c+1}^d(x)\right\}\right] \Big/ \sum_{j=1}^{\infty}\pi_{j,0}. \end{aligned} \tag{3}$$

2.4. Passenger waiting time at the lobby

We will find the waiting time W for a passenger. We define $X_{i,j}$ as the time period between a Markov point with state (i,j) and the next Markov point. Then,

$$X_{i,j} \equiv \begin{cases} \text{idle period at the } j\text{th floor} & \text{if } i=0, j>0, \\ \text{duration of down-trip from } j\text{th floor and stop time at the lobby} & \text{if } i>0, j>0, \\ \text{duration of up-trip serving } (i,c)^- \text{ passengers} & \text{if } i>0, j=0. \end{cases}$$

That is, if $i = 0$ and $j > 0$, then the probability distribution of $X_{i,j}$ is an exponential distribution with mean $1/\lambda$. If $i > 0$ and $j > 0$, then $X_{i,j}$ has the deterministic value $jt_r + t_l$. Finally, if $i > 0$ and $j = 0$, then $X_{i,j}$ has the following distribution:

$$P\{X_{i,j} = lt_r + mt_s\} = G_{(i,c)^-}(l, m), \quad l = 1, 2, \ldots, N, m = 1, 2, \ldots, (i, l, c)^-.$$

The mean $\eta_{i,j}$ of $X_{i,j}$ is given by

$$\eta_{i,j} = \begin{cases} 1/\lambda & \text{if } i = 0, j > 0, \\ jt_r + t_l & \text{if } i > 0, j > 0, \\ \sum_{l=1}^{N} \sum_{m=1}^{(i,l,c)^-} (lt_r + mt_s) G_{(i,c)^-}(l, m) & \text{if } i > 0, j = 0. \end{cases}$$

Now consider a virtual passenger that arrives at the lobby. The passenger may arrive during one of the following three time periods; up-trip, down-trip and idle period. With probability $\pi_{i,j}\eta_{i,j}/\sum_{k,l}\pi_{k,l}\eta_{k,l}$ the virtual passenger arrives during a time period $X_{i,j}$. The Laplace–Stieljes Transform(LST) for the conditional waiting time $W_{i,j}$ of the passenger arriving during a time period $X_{i,j}$ is denoted by $W^*_{i,j}(s)$. Then the conditional waiting time $W_{0,j}, j = 1, 2, \ldots, N$, is the duration of a down-trip from jth floor, i.e., $W_{0,j} = jt_r + t_l$, with LST

$$W^*_{0,j}(s) = \mathrm{e}^{-s(jt_r+t_l)}.$$

Next, consider the case that a virtual passenger arrives y time units before the end of a down-trip $X_{i,j}$, $i, j > 0$, during the down-trip period. Note that the duration of a down-trip $X_{i,j}$ is $jt_r + t_l$. If α denotes the number of passengers that arrive during $X_{i,j} - y$, then

$$E\left[\mathrm{e}^{-sW_{i,j}} | \text{ arrival at } y, \alpha = n\right] = \mathrm{e}^{-sy}[R^*_{c+1}(s)]^{\lceil (i+n)/c \rceil}, \qquad i, j > 0,$$

where $R^*_i(s), i = 1, 2, \ldots, c + 1$, denotes the LST of a type-i round-trip time R_i. Unconditioning on α yields

$$E\left[\mathrm{e}^{-sW_{i,j}} | \text{ arrival at } y\right] = \sum_{n=0}^{\infty} \mathrm{e}^{-sy}[R^*_{c+1}(s)]^{\lceil (i+n)/c \rceil} \frac{[\lambda(jt_r + t_l - y)]^n}{n!} \mathrm{e}^{-\lambda(jt_r+t_l-y)}$$
$$= \mathrm{e}^{-sy-\lambda(jt_r+t_l-y)} \sum_{n=0}^{\infty} \frac{[\lambda(jt_r + t_l - y)]^n}{n!} [R^*_{c+1}(s)]^{\lceil (i+n)/c \rceil}.$$

Since the arrival time y is uniformly distributed between 0 and $jt_r + t_l$, its probability density function is given by $\mathrm{d}y/(jt_r + t_l)$. Therefore, unconditioning on the arrival time y yields

$$W^*_{i,j}(s) = \frac{\mathrm{e}^{-s(jt_r+t_l)}}{jt_r + t_l} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} [R^*_{c+1}(s)]^{\lceil (i+n)/c \rceil} \int_0^{jt_r+t_l} y^n \mathrm{e}^{-(\lambda-s)y} \mathrm{d}y$$
$$= \frac{\mathrm{e}^{-s(jt_r+t_l)}}{(\lambda - s)(jt_r + t_l)} \sum_{n=0}^{\infty} \left(\frac{\lambda}{\lambda - s}\right)^n [R^*_{c+1}(s)]^{\lceil (i+n)/c \rceil} \left[1 - \sum_{k=0}^{n} \frac{[(\lambda - s)(jt_r + t_l)]^k}{k!} \mathrm{e}^{-(\lambda-s)(jt_r+t_l)}\right], \quad i, j > 0.$$

Finally, we consider the case that a virtual passenger arrives y time units before the end of an up-trip $X_{i,0}$, $i = 1, 2, \ldots$, during the up-trip period. If α denotes the number of passengers that arrive during $X_{i,0} - y$, then

$$E\left[\mathrm{e}^{-sW_{i,0}} | X_{i,0} = lt_r + mt_s, \text{ arrival at } y, \alpha = n\right] = \mathrm{e}^{-s(y+lt_r+t_l)}[R^*_{c+1}(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil}.$$

Unconditioning on α yields

$$E\left[\mathrm{e}^{-sW_{i,0}} | X_{i,0} = lt_r + mt_s, \text{ arrival at } y\right]$$
$$= \sum_{n=0}^{\infty} \mathrm{e}^{-s(y+lt_r+t_l)}[R^*_{c+1}(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} \frac{[\lambda(lt_r + mt_s - y)]^n}{n!} \mathrm{e}^{-\lambda(lt_r+mt_s-y)}$$
$$= \mathrm{e}^{-s(y+lt_r+t_l)-\lambda(lt_r+mt_s-y)} \sum_{n=0}^{\infty} \frac{[\lambda(lt_r + mt_s - y)]^n}{n!} [R^*_{c+1}(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil}, \quad i > 0.$$

Unconditioning on the arrival time y yields

$$E\left[\mathrm{e}^{-sW_{i,0}} | X_{i,0} = lt_r + mt_s\right] = \frac{\mathrm{e}^{-s(2lt_r+mt_s+t_l)}}{lt_r + mt_s} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} [R^*_{c+1}(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} \int_0^{lt_r+mt_s} y^n \mathrm{e}^{-(\lambda-s)y} \mathrm{d}y$$
$$= \frac{\mathrm{e}^{-s(2lt_r+mt_s+t_l)}}{(\lambda - s)(lt_r + mt_s)} \sum_{n=0}^{\infty} \left(\frac{\lambda}{\lambda - s}\right)^n [R^*_{c+1}(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil}$$
$$\times \left[1 - \sum_{k=0}^{n} \frac{[(\lambda - s)(lt_r + mt_s)]^k}{k!} \mathrm{e}^{-(\lambda-s)(lt_r+mt_s)}\right], \quad i > 0.$$

Furthermore, by the batch effect, the probability that the period $X_{i,0}$ during which the virtual passenger arrives is $lt_r + mt_s$ is given by $(lt_r + mt_s)G_{(i,c)^-}(l, m)/\eta_{i,0}$, $i > 0$. Therefore, by unconditioning on the length of $X_{i,0}$, we get the following LST

$W^*_{i,0}(s)$ for the waiting time of the virtual passenger that arrives during the up-trip $X_{i,0}$;

$$W^*_{i,0}(s) = \sum_{l=1}^{N}\sum_{m=1}^{(c,i,l)^-} E\left[e^{-sW_{i,0}}|X_{i,0} = lt_r + mt_s\right] \frac{(lt_r + mt_s)G_{(i,c)^-}(l,m)}{\eta_{i,0}}$$
$$= \frac{e^{-st_l}}{(\lambda - s)\eta_{i,0}} \sum_{l=1}^{N}\sum_{m=1}^{(c,i,l)^-} e^{-s(2lt_r+mt_s)} G_{(i,c)^-}(l,m) \sum_{n=0}^{\infty}\left(\frac{\lambda}{\lambda - s}\right)^n$$
$$\times [R^*_{c+1}(s)]^{\lceil\{i-(i,c)^-+n\}/c\rceil}\left[1 - \sum_{k=0}^{n}\frac{[(\lambda - s)(lt_r + mt_s)]^k}{k!} e^{-(\lambda-s)(lt_r+mt_s)}\right], \quad i > 0.$$

By unconditioning on the state of the Markov chain before the virtual passenger arrives, we get the LST for the waiting time of a virtual passenger;

$$W^*(s) = \sum_{i,j}\pi_{i,j}\eta_{i,j}W^*_{i,j}(s) \Big/ \sum_{k,l}\pi_{k,l}\eta_{k,l}. \tag{4}$$

2.5. Passenger ride time

After an elevator has arrived to serve a passenger, the ride time of the passenger begins when the passenger steps inside the elevator. The ride time ends when the passenger exits the elevator. During up-peak, when several passengers travel in the elevator, the passenger ride time consists of the elevator run time to the destination floor and the stop times before reaching the destination floor of a virtual passenger. Hence, it depends on the destination floor and the number of intermediate stops before the destination. With m stops on the way before getting to destination floor l, the ride time consists of the sum of lt_r and mt_s. The value lt_r refers to the run time to the destination floor l. The value mt_s refers to the sum of m stop times during the ride. Thus, the passenger ride time T takes the only values of the form $lt_r + mt_s$ for $l = 1, 2, \ldots, N$ and $m = 0, 1, \ldots, l-1$.

Let M be the number of passengers inside the elevator, to which a virtual passenger belongs, when it leaves the lobby. By the batch effect, we obtain

$$P\{M = i\} = \begin{cases} i\pi_{i,0}\Big/\left[c - \sum_{j=1}^{c-1}(c-j)\pi_{j,0}\right] & \text{if } i = 1, 2, \ldots, c-1, \\ c\left[1 - \sum_{j=1}^{c-1}\pi_{j,0}\right]\Big/\left[c - \sum_{j=1}^{c-1}(c-j)\pi_{j,0}\right] & \text{if } i = c. \end{cases}$$

We obtain, by considering all the possible stop combinations before reaching destination floor,

$$P\{T = lt_r + mt_s|M = i\} = \frac{1}{N^i}\binom{l-1}{m} \sum_{\substack{m+1\le g_1+\cdots+g_{m+1}\le i \\ 1\le g_1,\ldots,g_{m+1}\le i-m}} (N-l)^{\left(i-\sum_{e=1}^{m+1} g_e\right)} \left[\prod_{e=1}^{m+1}\binom{i - \sum_{a=1}^{e-1} g_a}{g_e}\right], \tag{5}$$

where 0^0 is assumed to be 1. Unconditioning on M yields

$$P\{T = lt_r + mt_s\} = \sum_{i=m+1}^{c} P\{M = i\}P\{T = lt_r + mt_s|M = i\} \tag{6}$$

for $l = 1, 2, \ldots, N$ and $m = 0, 1, \ldots, (l-1, c-1)^-$.

2.6. Passenger journey time

We will find the journey time, which is defined as the total time a passenger spends in an elevator system. Let us assume that arriving passengers form a queue in the order of arrivals in the lobby. Let M' denote the number of passengers who are served at the same time as the virtual passenger, under the condition that no more passengers are allowed to join the lobby.

We denote by $J^*(s|i,j)$ the LST of the remaining journey time of a virtual passenger with position i, given that $M' = j$ immediately before a round-trip time. Then,

$$J^*(s|i,j) = \begin{cases} T^*_j(s) \quad \text{if } i \le c, i \le j \le c, \\ T^*_c(s)[R^*_{c+1}(s)]^{i/c-1} \quad \text{if } i > c, i = 0 \bmod c, j = c \\ \sum_{k=0}^{c-j-1} R^*_{c+1}(k,s)\left[J^*(s|i-c, j+k) - [R^*_{c+1}(s)]^{\lceil i/c\rceil - 1}T^*_c(s)\right] + T^*_c(s)[R^*_{c+1}(s)]^{\lceil i/c\rceil} \\ \quad \text{if } i > c, i \ne 0 \bmod c, i - \lceil i/c\rceil c \le j \le c, \end{cases}$$

where

$$T_j^*(s) = E[e^{-sT}|M = j], \quad j = 1, 2, \ldots, c,$$

$$R_i^*(k, s) = \int_0^\infty e^{-sx} \frac{(\lambda x)^k e^{-\lambda x}}{k!} dR_i(x), \quad i = 1, 2, \ldots, c + 1, k = 0, 1, \ldots.$$

The LST for the conditional journey time $J_{i,j}$ of the passenger arriving during a time period $X_{i,j}$ is denoted by $J_{i,j}^*(s)$. Then the conditional journey time $J_{0,j}^*(s), j > 0$, is

$$J_{0,j}^*(s) = e^{-s(jt_r+t_l)} \left[T_c^*(s) + \sum_{k=0}^{c-2} \frac{[\lambda(jt_r + t_l)]^k}{k!} e^{-\lambda(jt_r+t_l)} \left\{ T_{k+1}^*(s) - T_c^*(s) \right\} \right], \quad j > 0.$$

Next consider the case that a virtual passenger arrives y time units before the end of a down-trip $X_{i,j}$, $i, j > 0$, during the down-trip period. If α denotes the number of passengers that arrive during $X_{i,j} - y$, then

$$E[e^{-sJ_{i,j}} | \text{ arrival at } y, \alpha = n] = e^{-sy} \left[[R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \right.$$
$$\left. + \sum_{k=0}^{c-\beta_d-2} \frac{(\lambda y)^k e^{-\lambda y}}{k!} \left\{ J^*(s|i + n + 1, \beta_d + 1 + k) - [R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \right\} \right], i, j > 0,$$

where

$$\beta_d = i + n - \left\lceil \frac{i + n}{c} \right\rceil c.$$

Unconditioning on α yields

$$E[e^{-sJ_{i,j}} | \text{ arrival at } y] = e^{-sy-\lambda(jt_r+t_l-y)} \sum_{n=0}^{\infty} \frac{[\lambda(jt_r + t_l - y)]^n}{n!} \left[[R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \right.$$
$$\left. + \sum_{k=0}^{c-\beta_d-2} \frac{(\lambda y)^k e^{-\lambda y}}{k!} \left\{ J^*(s|i + n + 1, \beta_d + 1 + k) - [R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \right\} \right], \quad i, j > 0.$$

Unconditioning on the arrival time y yields

$$J_{i,j}^*(s) = \sum_{n=0}^{\infty} \lambda^n \left[\frac{e^{-s(jt_r+t_l)}}{(jt_r + t_l)(\lambda - s)^{n+1}} [R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \left\{ 1 - \sum_{m=0}^{\infty} \frac{[(\lambda - s)(jt_r + t_l)]^m}{m!} e^{-(\lambda-s)(jt_r+t_l)} \right\} \right.$$
$$+ \frac{(jt_r + t_l)^{n-1}}{s} e^{-\lambda(jt_r+t_l)} \sum_{k=0}^{c-\beta_u-2} \left(\frac{\lambda}{s} \right)^k \frac{1}{k!} \left\{ J^*(s|i + n + 1, \beta_d + 1 + k) - [R_{c+1}^*(s)]^{\lceil (i+n)/c \rceil} T_c^*(s) \right\}$$
$$\left. \times \sum_{l=0}^{n} \frac{(-1)^l (k + l)!}{[s(jt_r + t_l)]^l l!(n - l)!} \left\{ 1 - \sum_{m=0}^{k+l} \frac{[s(jt_r + t_l)]^m}{m!} e^{-s(jt_r+t_l)} \right\} \right], \quad i, j > 0.$$

Finally, we consider the case that a virtual passenger arrives y time units before the end of an up-trip $X_{i,0}$, $i > 0$, during the up-trip period. If α denotes the number of passengers that arrive during $X_{i,0} - y$, then

$$E[e^{-sJ_{i,0}} | X_{i,0} = lt_r + mt_s, \text{ arrival at } y, \alpha = n] = e^{-s(y+lt_r+t_l)} \left[[R_{c+1}^*(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} T_c^*(s) \right.$$
$$\left. + \sum_{k=0}^{c-\beta_u-2} \frac{(\lambda y)^k e^{-\lambda y}}{k!} \left\{ J^*(s|i - (i, c)^- + n + 1, \beta_u + 1 + k) - [R_{c+1}^*(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} T_c^*(s) \right\} \right],$$

where

$$\beta_u = i - (i, c)^- + n - \left\lceil \frac{i - (i, c)^- + n}{c} \right\rceil c.$$

Unconditioning on α yields

$$E[e^{-sJ_{i,0}} | X_{i,0} = lt_r + mt_s, \text{ arrival at } y]$$
$$= e^{-s(y+lt_r+t_l)-\lambda(lt_r+mt_s-y)} \sum_{n=0}^{\infty} \frac{[\lambda(lt_r + mt_s - y)]^n}{n!} \left[[R_{c+1}^*(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} T_c^*(s) \right.$$
$$\left. + \sum_{k=0}^{c-\beta_u-2} \frac{(\lambda y)^k e^{-\lambda y}}{k!} \left\{ J^*(s|i - (i, c)^- + n + 1, \beta_u + 1 + k) - [R_{c+1}^*(s)]^{\lceil \{i-(i,c)^- + n\}/c \rceil} T_c^*(s) \right\} \right].$$

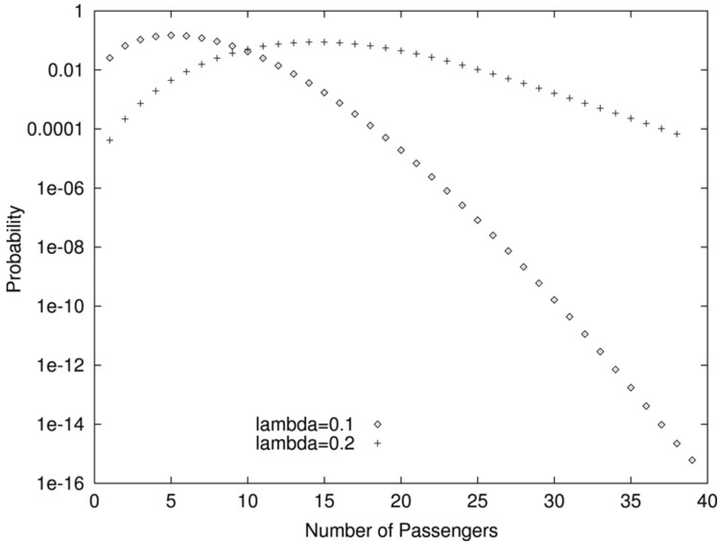

Fig. 1. Probability mass function of the number of passengers at the lobby.

Unconditioning on the arrival time y yields

$$\begin{aligned}
E[\mathrm{e}^{-sJ_{i,0}}|X_{i,0}=lt_r+mt_s] &= \sum_{n=0}^{\infty}\lambda^n\left[\frac{\mathrm{e}^{-s(2lt_r+mt_s+t_l)}}{(lt_r+mt_s)(\lambda-s)^{n+1}}[R^*_{c+1}(s)]^{\lceil\{i-(i,c)^-+n\}/c\rceil}T^*_c(s)\right.\\
&\times\left\{1-\sum_{q=0}^{\infty}\frac{[(\lambda-s)(lt_r+mt_s)]^q}{q!}\mathrm{e}^{-(\lambda-s)(lt_r+mt_s)}\right\}+\frac{(lt_r+mt_s)^{n-1}}{s}\mathrm{e}^{-s(lt_r+t_l)-\lambda(lt_r+mt_s)}\\
&\times\sum_{k=0}^{c-\beta_u-2}\left(\frac{\lambda}{s}\right)^k\frac{1}{k!}\left\{J^*(s|i-(i,c)^-+n+1,\beta_u+1+k)-[R^*_{c+1}(s)]^{\lceil\{i-(i,c)^-+n\}/c\rceil}T^*_c(s)\right\}\\
&\left.\times\sum_{p=0}^{n}\frac{(-1)^p(k+p)!}{[s(lt_r+mt_s)]^p p!(n-p)!}\left\{1-\sum_{q=0}^{k+p}\frac{[s(lt_r+mt_s)]^q}{q!}\mathrm{e}^{-s(lt_r+mt_s)}\right\}\right],\quad i,j>0.
\end{aligned}$$

Unconditioning on the length of $X_{i,0}$, we get

$$\begin{aligned}
J^*_{i,0}(s) &= \sum_{l=1}^{N}\sum_{m=1}^{(c,i,l)^-}E[\mathrm{e}^{-sJ_{i,0}}|X_{i,0}=lt_r+mt_s]\frac{(lt_r+mt_s)G_{(i,c)^-}(l,m)}{\eta_{i,0}}\\
&= \frac{\mathrm{e}^{-st_l}}{\eta_{i,0}}\sum_{l=1}^{N}\sum_{m=1}^{(c,i,l)^-}(lt_r+mt_s)G_{(i,c)^-}(l,m)\sum_{m=0}^{\infty}\lambda^n\left[\frac{\mathrm{e}^{-s(2lt_r+mt_s+t_l)}}{(lt_r+mt_s)(\lambda-s)^{n+1}}\right.\\
&\quad\times[R^*_{c+1}(s)]^{\lceil\{i-(i,c)^-+n\}/c\rceil}T^*_c(s)\left\{1-\sum_{q=0}^{\infty}\frac{[(\lambda-s)(lt_r+mt_s)]^q}{q!}\mathrm{e}^{-(\lambda-s)(lt_r+mt_s)}\right\}\\
&\quad+\frac{(lt_r+mt_s)^{n-1}}{s}\mathrm{e}^{-s(lt_r+t_l)-\lambda(lt_r+mt_s)}\\
&\quad\times\sum_{k=0}^{c-\beta_u-2}\left(\frac{\lambda}{s}\right)^k\frac{1}{k!}\left\{J^*(s|i-(i,c)^-+n+1,\beta_u+1+k)-[R^*_{c+1}(s)]^{\lceil\{i-(i,c)^-+n\}/c\rceil}T^*_c(s)\right\}\\
&\quad\left.\times\sum_{p=0}^{n}\frac{(-1)^p(k+p)!}{[s(lt_r+mt_s)]^p p!(n-p)!}\left\{1-\sum_{q=0}^{k+p}\frac{[s(lt_r+mt_s)]^q}{q!}\mathrm{e}^{-s(lt_r+mt_s)}\right\}\right],\quad i,j>0.
\end{aligned}$$

By unconditioning on the state of the Markov chain before the virtual passenger arrives, we get the LST for the journey time of the virtual passenger:

$$J^*(s)=\sum_{i,j}\pi_{i,j}\eta_{i,j}J^*_{i,j}(s)/\sum_{k,l}\pi_{k,l}\eta_{k,l}. \tag{7}$$

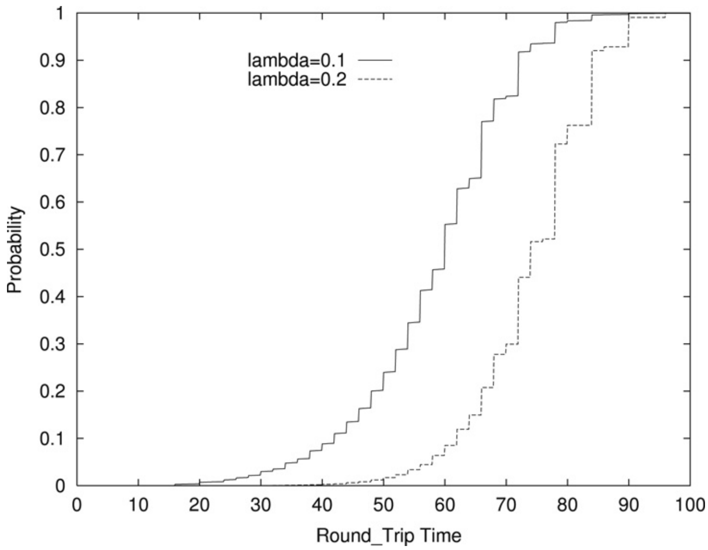

Fig. 2. Cumulative distribution function of the round-trip time.

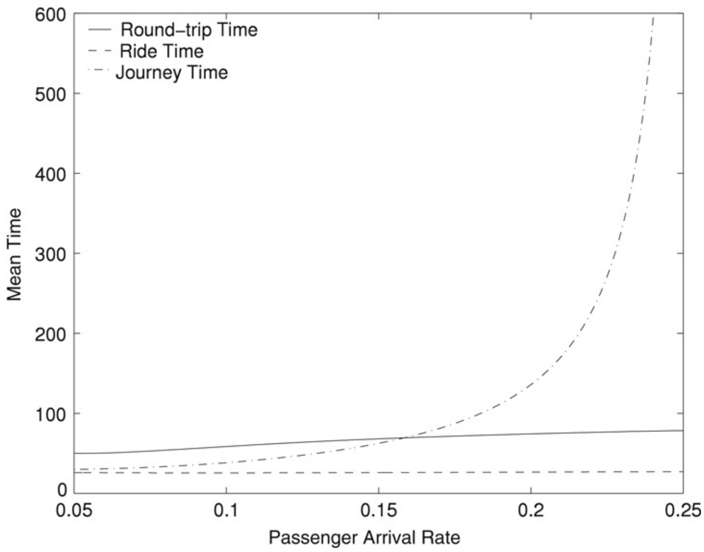

Fig. 3. Mean of the round-trip time, the ride time and the journey time.

3. Numerical examples

In this section, some numerical examples are presented for an elevator system of 9 floors above the lobby, an elevator capacity of 20 passengers, a constant inter-floor travel time of 2 s, and a constant stop time at a floor of 6 s. These parameters are solely for illustration purposes and can be modified to reflect other situations. In the example, all the passengers arrive randomly at the lobby according to Poisson process, with rate λ, and travel to one of the higher floors, chosen randomly as their destination. After serving all passengers, the elevator returns to the lobby if there are passengers waiting at the lobby; otherwise, the elevator stays idle at the highest destination floor and is activated by the first passenger arrival at the lobby.

Fig. 1 displays the probability mass function of the number of passengers in the elevator system (for different values of passenger arrival rate) at the departure epoch from the lobby. As we expected, the tail of the number of passengers with higher arrival rate $\lambda = 0.2$ is longer than that with lower arrival rate $\lambda = 0.1$.

Fig. 2 displays the cumulative distribution function of the round-trip time for $\lambda = 0.1$ and $\lambda = 0.2$. The round-trip time for $\lambda = 0.1$ are shorter than that for $\lambda = 0.2$. Furthermore, when $\lambda = 0.1$ and $\lambda = 0.2$, the continuous part of the round-trip time due to the idle time, if any, of the elevator at the highest reversal floor does not have much effect on the total round-trip time.

Fig. 3 displays the mean round-trip time, ride time and journey time as a function of the arrival rate. The mean round-trip time and the mean ride time increase slowly with increasing arrival rate. The mean journey time, which is obtained by calculating derivatives of the LST for the journey time at zero, increases rapidly as the arrival rate increases. Thus, as we mentioned in Section 1, the conventional assumption that the waiting time, which is the difference between the journey time and the ride time, is, at most, the round-trip time, is non-realistic.

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References

[1] G.C. Barney, S.M. dos Santos, Improved traffic design methods for lift systems, Building Sci. 10 (4) (1975) 277–285.
[2] G.C. Barney, S.M. dos Santos, Elevator Traffic Analysis, Design and Control, Peregrinus, London, 1985.
[3] B.W. Jones, Simple analytical method for evaluating lift performance during up peak, Transportation Research Part B: Methodological 5 (4) (1971) 301–307.
[4] D. Levy, M. Yadin, A. Alexsandrovitz, Optimal control of elevators, International Journal Systems Science 8 (3) (1977) 301–320.
[5] P.R. Tregenza, The prediction of lift performance in multi-story buildings, Ph.D. Thesis, Department of Architecture, University of Nottingham, UK, 1971.
[6] T. Nagatani, Complex behavior of elevators in peak traffic, Physica A 326 (2003) 556–566.
[7] M. Siikonen, Customer service an elevator system during up-peak, Transportation Research Part B: Methodological 31 (2) (1997) 127–139.
[8] G.F. Newell, Strategies for serving peak elevator traffic, Transportation Research Part B: Methodological 32 (8) (1998) 583–588.
[9] N.A. Alexandris, C.J. Harris, G.C. Barney, Evaluation of the handling capacity of multi-car lift system analysis and design, Applied Mathematical Modelling 5 (2) (1981) 49–52.
[10] N.R. Roschier, M.J. Kaakinen, New formulae for Elevator round-trip calculation, Supplement to *Elevator World* for ACIST member (1979) 189–197.
[11] M. Siikonen, Elevator traffic simulation, Simulation 61 (4) (1993) 257–267.
[12] N.A. Alexandris, Statistical models in lift systems, Ph.D. Thesis, Control Systems Centre, Institute of Science and Technology, University of Manchester, Manchester, UK, 1977.
[13] M.F. Neuts, Structured Stochastic Matrices of M/G/1 Type and their Applications, Marcel Dekkar, New York, 1989.